

QGP collective effects and jet transport

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Abstract. We present numerical simulations of the SU(2) Boltzmann-Vlasov equation including both hard elastic particle collisions and soft interactions mediated by classical Yang-Mills fields. We provide an estimate of the coupling of jets to a hot isotropic plasma which is independent of infrared cutoffs. In addition, we investigate jet propagation in anisotropic plasmas, as created in heavy-ion collisions. The broadening of jets is found to be stronger along the beam line than in azimuth, due to the creation of field configurations with $B_{\perp} > E_{\perp}$ and $E_z > B_z$ via plasma instabilities.

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1. Introduction

High transverse momentum jets produced in heavy-ion collisions represent a valuable tool for studies of the properties of the hot parton plasma produced in the central rapidity region [1]. However, present estimates of the strength of the coupling of jets to a QCD plasma are sensitive to infrared cutoffs. We employ a numerical simulation of the Boltzmann-Vlasov equation, which is coupled to the Yang-Mills equation for the soft gluon degrees of freedom. Soft momentum exchanges between particles are mediated by the fields, while hard momentum exchanges are described by a collision term including binary elastic collisions. This way, we are able to provide an estimate of the coupling of jets to a hot plasma which is independent of infrared cutoffs.

The longitudinal expansion of the plasma may lead to a strongly anisotropic momentum distribution in the local rest frame, during the very early stages of the plasma evolution. Due to anisotropies in the particle momentum distributions plasma instabilities appear [2]. These lead to the formation of long-wavelength chromo-fields with $E_z > B_z$ and $B_{\perp} > E_{\perp}$, which affect the propagation of a hard jet and of its induced hard radiation field. This may provide an explanation for the observed asymmetry in measurements of dihadron-correlations. Here a much stronger broadening of jets in pseudorapidity (η) than in azimuthal angle (ϕ) has been observed [1, 3].

2. Boltzmann-Vlasov equation for non-Abelian gauge theories

We solve the classical transport equation for hard gluons with SU(2) color charge including hard binary collisions

$$p^\mu [\partial_\mu + gq^a F_{\mu\nu}^\nu \partial_p^\nu + gf^{abc} A_\mu^b(x) q^c \partial_{q^a}] f = \mathcal{C}, \quad (1)$$

where $f = f(x, p, q)$ denotes the single-particle phase space distribution. It is coupled self-consistently to the Yang-Mills equation for the soft gluon fields. The collision term contains all binary collisions, described by the leading-order $gg \rightarrow gg$ tree-level diagrams.

We replace the distribution $f(x, p, q)$ by a large number of test particles, which leads to Wong's equations [4]

$$\begin{aligned} \dot{\mathbf{x}}_i(t) &= \mathbf{v}_i(t), \quad \dot{\mathbf{p}}_i(t) = gq_i^a(t) (\mathbf{E}^a(t) + \mathbf{v}_i(t) \times \mathbf{B}^a(t)), \\ \dot{q}_i(t) &= -igv_i^\mu(t) [A_\mu(t), q_i(t)], \end{aligned} \quad (2)$$

for the i -th test particle, whose coordinates are $\mathbf{x}_i(t)$, $\mathbf{p}_i(t)$, and $q_i^a(t)$. The time evolution of the Yang-Mills field is determined by the standard Hamiltonian method [5] in $A^0 = 0$ gauge. See [6, 7, 8, 9] for more details. The collision term is incorporated using the stochastic method [10]. The total cross section is given by $\sigma_{2 \rightarrow 2} = \int_{k^*}^{s/2} \frac{d\sigma}{dq^2} dq^2$, where we have introduced a lower cutoff k^* . To avoid double-counting, this cutoff should be on the order of the hardest field mode that can be represented on the given lattice, $k^* \simeq \pi/a$, with the lattice spacing a . For a more detailed discussion see [9].

3. Jet broadening in an isotropic plasma

We first consider a heat-bath of particles with a density of $n_g = 10/\text{fm}^3$ and an average particle momentum of $3T = 12$ GeV. For a given lattice (resp. k^*) we take the initial energy density of the thermalized fields to be $\int d^3k/(2\pi)^3 k \hat{f}_{\text{Bose}}(k) \Theta(k^* - k)$, where $\hat{f}_{\text{Bose}}(k) = n_g/(2T^3 \zeta(3))/(e^{k/T} - 1)$ is a Bose distribution normalized to the assumed particle density n_g , and ζ is the Riemann zeta function. The initial spectrum is fixed to Coulomb gauge and $A_i \sim 1/k$. We measure the momentum broadening $\langle p_\perp^2 \rangle(t)$ of high-energy test particles ($p/3T \approx 5$) passing through this medium. Fig. 1 shows that in the collisionless case, $\mathcal{C} = 0$, the broadening is stronger on larger lattices which accommodate harder field modes. However, Fig. 2 demonstrates that collisions with momentum exchange larger than $k^*(a)$ compensate for this growth and lead to approximately lattice-spacing independent results.

A related transport coefficient is \hat{q} [11]. It is the typical momentum transfer (squared) per collision divided by the mean-free path, which is nothing but $\langle p_\perp^2 \rangle(t)/t$. From Fig. 2, $\hat{q} \simeq 2.2$ GeV²/fm for $N_c = 2$, $n_g = 10/\text{fm}^3$ and $p/(3T) \approx 5$. We have verified that \hat{q} does not depend on the temperature T as long as the particle density n_g and the ratio p/T is fixed. Due to the independence of \hat{q} of the temperature and its proportionality to the density n , we can scale to physical densities for a QGP created at RHIC. We adjust for the different color factors in SU(3), and find $\hat{q} \approx 5.6$ GeV²/fm, at $T = 400$ MeV, $E_{\text{jet}} \approx 20$ GeV ($p/3T = 16$) in a system of quarks and gluons.

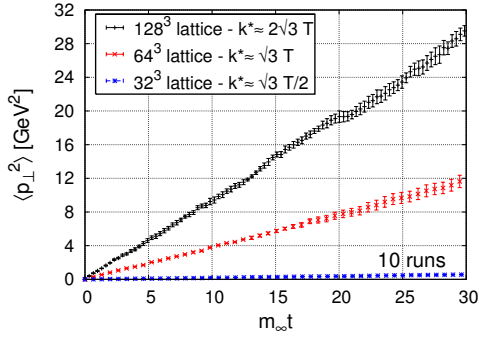


Figure 1. Momentum diffusion caused by particle-field interactions only.

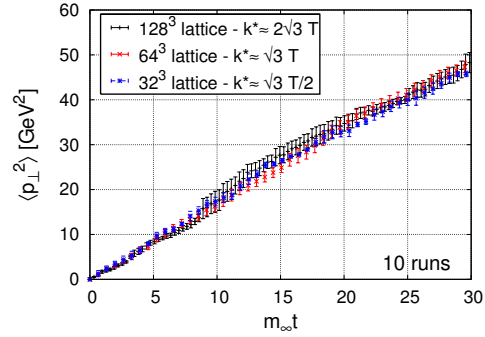


Figure 2. Momentum diffusion by both particle-field and direct particle-particle interactions.

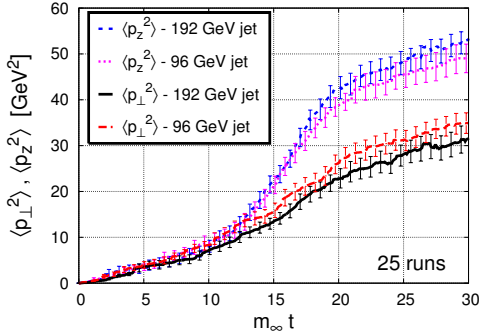


Figure 3. Momentum broadening of a jet transverse to its initial momentum.

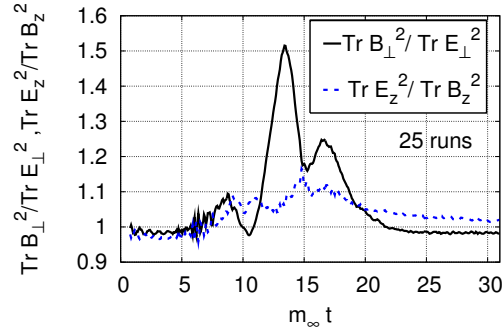


Figure 4. Ratios of field energy densities.

4. Jet broadening in an unstable plasma

In heavy-ion collisions, locally anisotropic momentum distributions may emerge due to the longitudinal expansion. Such anisotropies generically give rise to instabilities [2, 7, 8]. Here, we investigate their effect on the momentum broadening of jets, including the effect of collisions. The initial anisotropic momentum distribution for the hard plasma gluons is taken to be $f(\mathbf{p}) = n_g \left(\frac{2\pi}{p_h} \right)^2 \delta(p_z) \exp(-p_\perp/p_h)$, with $p_\perp = \sqrt{p_x^2 + p_y^2}$. We initialize small-amplitude fields sampled from a Gaussian distribution and set $k^* \approx p_h$, for the reasons alluded to above. We add additional high momentum particles with $p_x = 12 p_h$ and $p_x = 6 p_h$, respectively, to investigate the broadening in the y and z directions via the variances $\kappa_\perp(p_x) := \frac{d}{dt} \langle (\Delta p_\perp)^2 \rangle$, $\kappa_z(p_x) := \frac{d}{dt} \langle (\Delta p_z)^2 \rangle$. The ratio κ_z/κ_\perp can be roughly associated with the ratio of jet correlation widths in azimuth and rapidity: $\kappa_z/\kappa_\perp \approx \langle \Delta \eta \rangle / \langle \Delta \phi \rangle$. Experimental data on dihadron correlation functions for central Au+Au collisions at $\sqrt{s} = 200$ GeV [1] are consistent with $\kappa_z/\kappa_\perp \approx 3$ [12]. Fig. 3 shows the time evolution of $\langle p_\perp^2 \rangle$ and of $\langle p_z^2 \rangle$. During the period of instability and for both jet energies we find $\kappa_z/\kappa_\perp \approx 2.3$. The explanation for the larger broadening along

the beam axis is as follows. In the Abelian case the instability generates predominantly transverse magnetic fields which deflect the particles in the z -direction [13]. Although the interactions are a lot less trivial, in a non-Abelian plasma the instability creates large domains of strong chromo-electric and -magnetic fields with $E_z > B_z$, aside from $B_\perp > E_\perp$ (Fig. 4). The field configurations are such that particles are deflected preferentially in the longitudinal z -direction (to restore isotropy). Fig. 5 shows the filamentation of the current and the domains of magnetic fields generated by the instability.

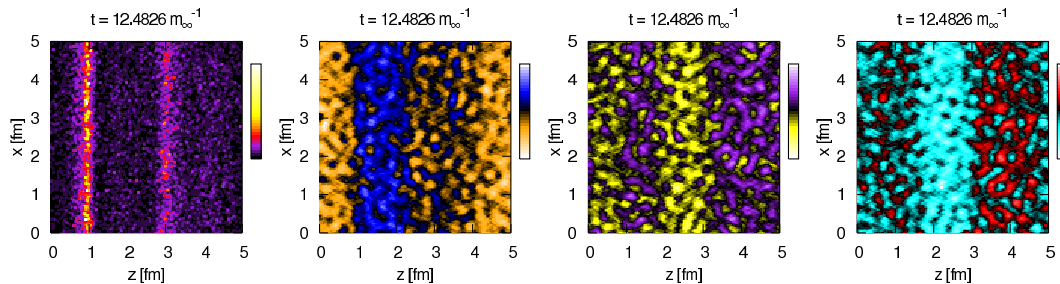


Figure 5. Slices in the x - z -plane at fixed $y = L/2$ of the current in the x -direction, J_x , and the three color components of the chromo-magnetic field in the y -direction. Filaments are nicely visible. Scales in lattice units. 0 to $5 \cdot 10^{-8}$ for the current, $-4 \cdot 10^{-3}$ to $4 \cdot 10^{-3}$ for the chromo-magnetic fields.

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